Math 3 Name Key

Final Exam Review-All work must be shown on a separate sheet of paper if you expect me to grade it!

1) Given: $f(x) = 2x - 3$ and $g(x) = x^2 - 5$, evaluate $3f(-3) + g(2)$ $3(2(-3)-3)+(2^2-5)=-27+-1=-2$

2) Given: $f(x) = \begin{cases} x^2+3 & \text{if } x < -2 \\ |x-1| & \text{if } -2 \le x < 3 \\ 5 & \text{if } x \ge 3 \end{cases}$, evaluate $3f(-1) + 2f(-3) - f(4)$.

$3(|-1-1|)+2((-3)^2+3)-5$
$= 6 + 24 - 5 = 25$

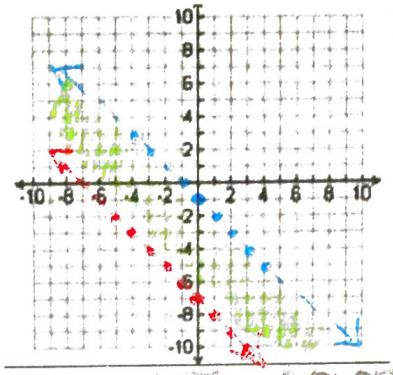
3) Use the factor theorem to show $(x-1)$ *and* $(x+2)$ are factors of $f(x) = 2x^4 + x^3 + 7x - 10$

$f(1) = 2(1)^4 + (1)^3 + 7(1) - 10 = 0$ $f(-2) = 2(-2)^4 + (-2)^3 + 7(-2) - 10 = 0$

4) Graph the system of inequalities.

$$\begin{cases} x + y < -1 \\ x + y \ge -7 \end{cases}$$

① $y < -x - 1$ ② $y \ge -x - 7$



5) Determine the inverse function of $f(x) = \sqrt{x} - 5$.

$\sqrt{x} - 5$ $+5$ $f^{-1}(x) = (x+5)^2$
D: $x \ge -5$

6) Simplify each of the following:

a. $(x^3 + 9x^2 - x - 105) \div (x + 5)$

b. $\dfrac{3r^2 + 27r}{r^2 + 7r - 18}$

c. $\dfrac{3n+18}{n^2+7n-30} \div \dfrac{n+6}{9n-27}$

d. $\log_3 27 + \log_2 16$

e. $(x^2 - 2x + 4)(x + 2)$

On separate paper

7) Solve each equation for x.

a. $|7 - 9k| = 43$

b. $\log_8(3x + 7) = \log_8(7x + 4)$

c. $7e^{2x} + 2.5 = 20$ (solve graphically)

d. $\dfrac{1}{x^2 + 4x} + \dfrac{x+3}{x+4} = \dfrac{1}{x}$

e. $4x^2 - 9 = 0$

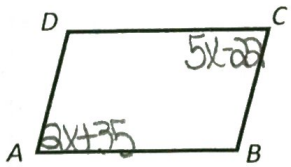
f. $2^{7x} - 1 = 58$

g. $0 = 2x^3 + 3x^2 - 39x - 20$ , $x = -5$ *is a solution*

h. $4^{2x+1} = 8^{2x}$

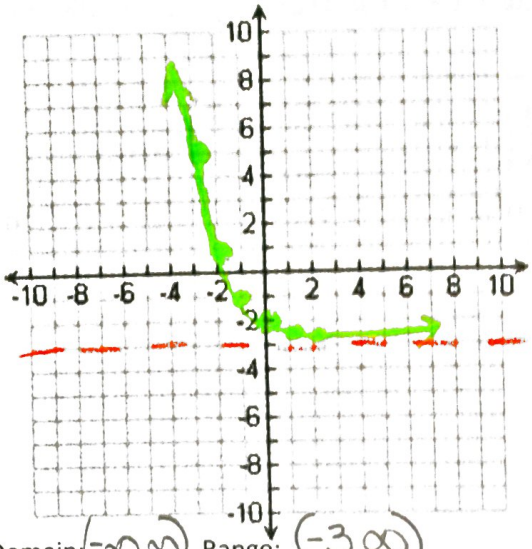
8) In the accompanying figure, *ABCD*, is a parallelogram $m\angle A = (2x + 35)°$ and $m\angle C = (5x - 22)°$. Determine the value of $x$.

$2x + 35 = 5x - 22$
$57 = 3x$
$19 = x$



9) Graph each function and fill in the information:

a. $y = \frac{1}{2}^x - 3$



Domain: $(-\infty, \infty)$ Range: $(-3, \infty)$

y-intercept: $(0, -2)$

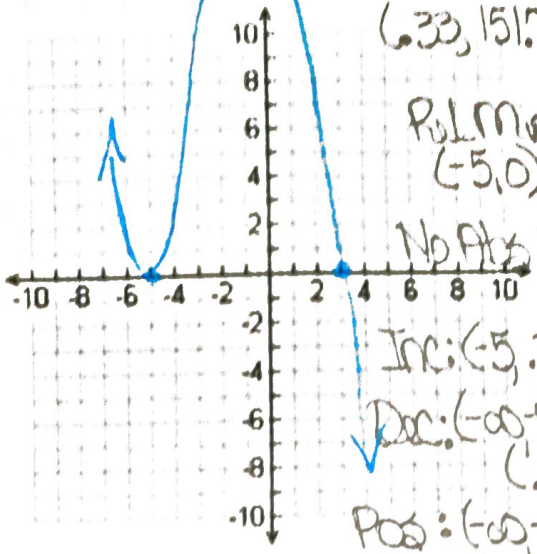
Asymptote: $y = -3$

Growth or Decay? Decay $(0 < b < 1)$

Initial Value: $1$

Growth/Decay Factor: $\frac{1}{2}$

*b. $y = -2(x-3)(x+5)^2$



Rel. Max. $(.33, 151.70)$

Rel. Min $(-5, 0)$

No Abs. max/min

Inc: $(-5, .33)$

Dec: $(-\infty, -5) \cup (.33, \infty)$

Pos: $(-\infty, -5) \cup (-5, 3)$

Neg.: $(3, \infty)$

Degree: $3$

# of Zeros: $3$ (2 distinct) # of Possible Max/Min. $2$

as $x \to -\infty$ $f(x) \to \infty$ as $x \to \infty$ $f(x) \to -\infty$

c. Graph and Identify Key Characteristics.

$y = \frac{x+2}{2x+6}$

Factored Form: $\frac{x+2}{y = 2(x+3)}$

Vertical Asymptote: $x = -3$

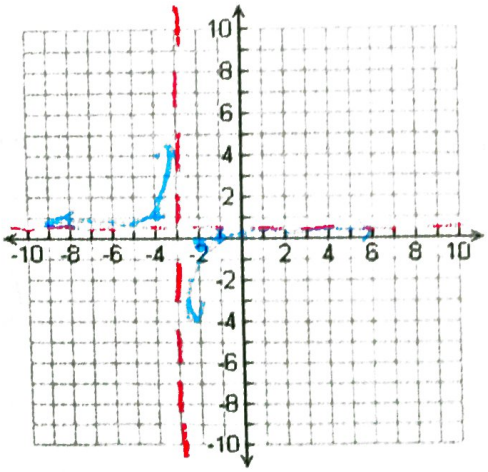
Horizontal Asymptote: $y = 1/2$

x-intercept(s): $(-2, 0)$ y-intercept: $(0, 1/3)$

Domain: $(-\infty, -3) \cup (-3, \infty)$

As $x \to -\infty$, $f(x) \to 1/2$

As $x \to \infty$, $f(x) \to 1/2$

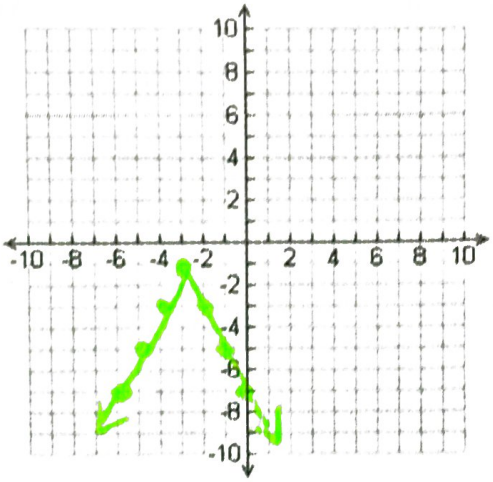


d. $f(x) = -2|x+3| - 1$

Vertex: $(-3, -1)$

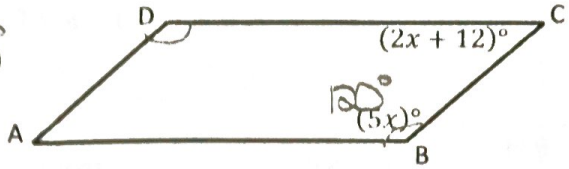
Domain: $(-\infty, \infty)$

Range: $(-\infty, -1]$



10) In the accompanying diagram of parallelogram $ABCD$, $m\angle B = (5x)°$ and $m\angle C = (2x + 12)°$. Determine the number of degrees in $m\angle D$.

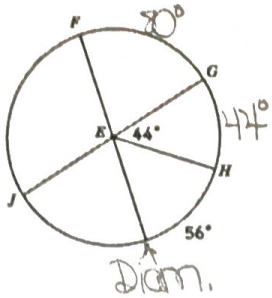
$m\angle D = 120°$



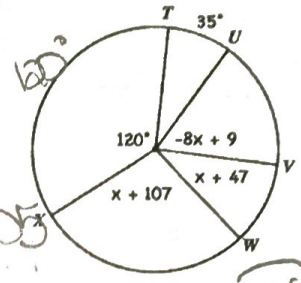

$2x+12+5x=180$
$7x=168$
$x=24$

**Determine the Arc or Angle Measures:**

11) $m\widehat{HF}$ = 128°



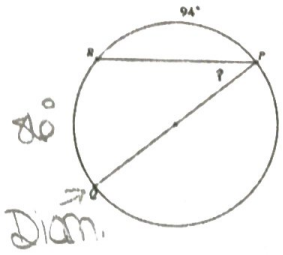

12) $m\widehat{VW}$ = 40°



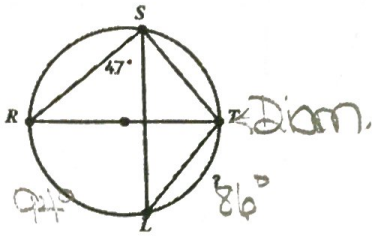

$x+107+x+47-8x+9=205$
$-6x+163=205$
$-6x=42$
$x=-7$

$m\widehat{XWSU}=205°$

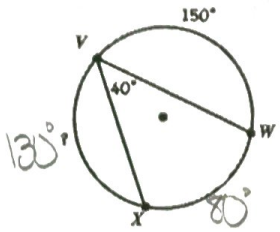
13) $m\angle QPR$ = 43°




14) $m\widehat{TL}$ = 86°

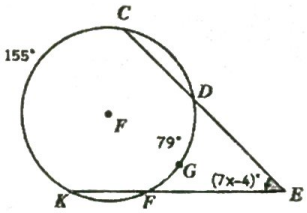



15) $m\widehat{VX}$ = 130°




16) x = 12

$7x-4=\frac{1}{2}(155-79)$
$7x-4=38$
$7x=42$




17) Identify the center and radius given the following equation:

a. $(x-4)^2 + (y-4)^2 = 3$

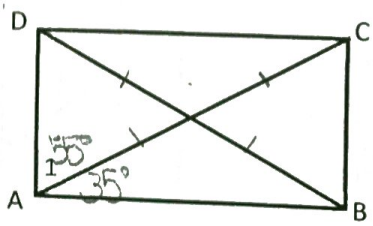
C: (4,4) $r=\sqrt{3}=1.73$

b. $(x+1)^2 + (y-2)^2 = 25$

C: (-1,2) $r=5$

18) In rectangle $ABCD$, $\overline{AC}$ and $\overline{BD}$ are diagonals. If $m\angle 1 = 55°$, determine $m\angle ABD$.

$m\angle ABD = 35°$



19) Fill in the table below using the information given. $0° \le \theta \le 90°, 0 \le \theta \le \frac{\pi}{2}$ *(1 pt each)*

| | $\theta$ in Degrees | $\theta$ in Radians | Sin $\theta$ | Cos $\theta$ |
|---|---|---|---|---|
| a. | 45° | $\frac{\pi}{4}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{2}}{2}$ |
| b. | 30° | $\frac{\pi}{6}$ | $\frac{1}{2}$ | $\frac{\sqrt{3}}{2}$ |
| c. | 30° | $\frac{\pi}{6}$ | $\frac{1}{2}$ | $\frac{\sqrt{3}}{2}$ |
| d. | 0° | 0 | 0 | 1 |
| e. | 45° | $\frac{\pi}{4}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{2}}{2}$ |

?

20) Evaluate the following trigonometric functions. *(3 pts. each)*

a. $\cos\frac{7\pi}{4} = \frac{\sqrt{2}}{2}$ (45°)

b. $\sin 180° =$ 0

c. $\sin\frac{\pi}{2} =$ 1

d. $\cos\frac{13\pi}{6} = \frac{\sqrt{3}}{2}$ (30°)

e. $\cos\frac{4\pi}{3} = -\frac{1}{2}$ (60°)

f. $\sin 150° = \frac{1}{2}$ (30°)

21) $f(\theta) = 3\sin\left(\frac{1}{2}\theta\right) - 1$

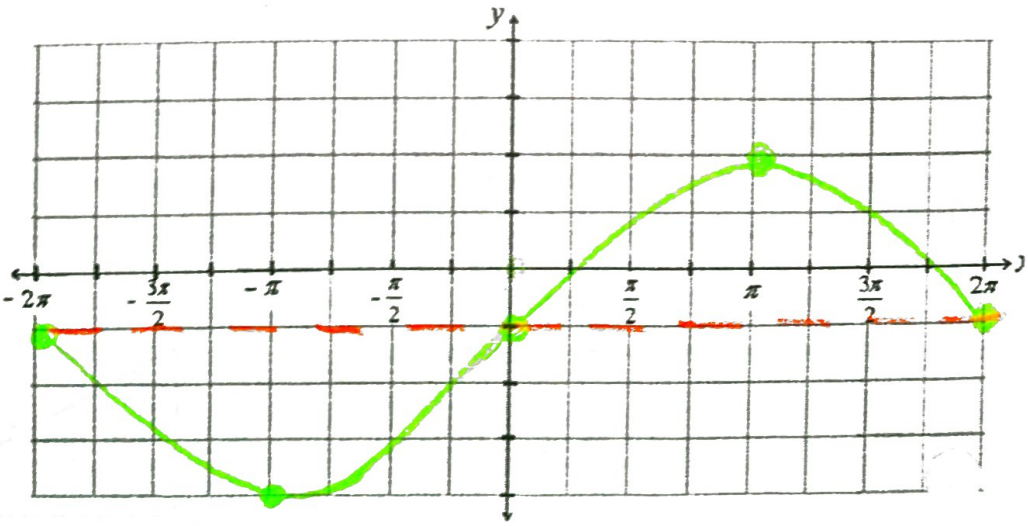
Amplitude: 3

Period: $4\pi$

Vertical Shift: down 1

Midline $y = -1$

Period $= \frac{2\pi}{\frac{1}{2}} = 4\pi$



# Exam Review

6) a)

$$\begin{array}{r|rrrr} -5 & 1 & 9 & -1 & -105 \\ & \downarrow & -5 & -20 & 105 \\ \hline & 1 & 4 & -21 & 0 \end{array}$$

$(x^3+9x^2-x-105) \div (x+5) = x^2+4x-21$

b) $\dfrac{3r^2+27r}{r^2+7r-18} = \dfrac{3r\cancel{(r+9)}}{\cancel{(r+9)}(r-2)} = \dfrac{3r}{r-2}$

c) $\dfrac{3n+18}{n^2+7n-30} \div \dfrac{n+6}{9n-27} = \dfrac{3\cancel{(n+6)}}{(n+10)\cancel{(n-3)}} \cdot \dfrac{9\cancel{(n-3)}}{\cancel{n+6}} = \dfrac{27}{n+10}$

d) $\log_3 27 + \log_2 16 = 3+4 = 7$

e) $(x^2-2x+4)(x+2) = x^3+\cancel{2x^2}-\cancel{2x^2}-\cancel{4x}+\cancel{4x}+8 = x^3+8$

7) a) $|7-9k| = 43$

$7-9k = 43$ or $7-9k = -43$

$-9k = 36$ $\quad$ $-9k = -50$

$k = -4$ $\quad$ $k = 50/9$

b) $\log_8(3x+7) = \log_8(7x+4)$

$3x+7 = 7x+4$

$3 = 4x$

$\frac{3}{4} = x$

c) $7e^{2x}+2.5 = 20$ (solves graphically)

$x = .458$

d) $\dfrac{1}{x^2+4x} + \dfrac{x(x+3)}{x(x+4)} = \dfrac{1}{x}$

$\dfrac{x^2+3x+1}{x^2+4x} = \dfrac{1}{x}$

$x^2+4x = x^3+3x^2+x$

$0 = x^3+2x^2-3x$

$0 = x(x^2+2x-3)$

$0 = x(x+3)(x-1)$

$\cancel{x=0}$, $x=-3$, $x=1$

e) $4x^2-9=0$

$(2x+3)(2x-3)=0$

$x = -\frac{3}{2}$, $x = \frac{3}{2}$

f) $2^{7x} - 1 = 58$

$2^{7x} = 59$

$\log_2 59 = 7x$

$.840 = x$

g) $\underline{-5}$ | 2 3 −39 −20

| | | | |
|---|---|---|---|
| ↓ | −10 | 35 | 20 |
| 2 | −7 | −4 | 0 |

$2x^2 - 7x - 4 = 0$

$2x^2 - 8x + 1x - 4 = 0$

$2x(x-4) + 1(x-4) = 0$

$(2x+1)(x-4) = 0$

$x = -\frac{1}{2},\ x = 4,\ x = -5$

h) $4^{2x+1} = 8^{2x}$

$(2^2)^{2x+1} = (2^3)^{2x}$

$2^{4x+2} = 2^{6x}$

$4x + 2 = 6x$

$2 = 2x$

$1 = x$