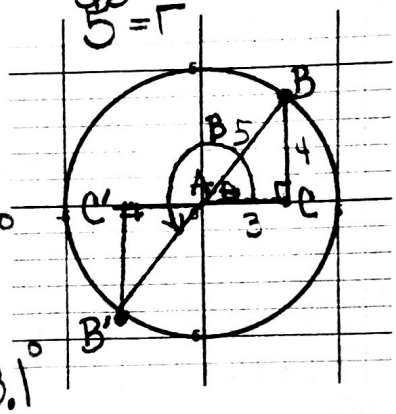
Key

NC Math 3
Unit 8 Review ~ Modeling Periodic Behavior

$3^2+4^2=r^2$
$25=r^2$
$5=r$

1. The diagram at the right shows two angles of rotation, θ and β, drawn in standard position. $\Delta A'B'C'$ is a 180° rotation of ΔABC about the origin.



pt B: (3, 4)

a. Show how you could use the sine ratio (SOH) to find the measure of angle θ to the nearest tenth of a degree.

$\sin\theta = \frac{opp}{hyp} = \frac{4}{5}$ $\theta = \sin^{-1}\left(\frac{4}{5}\right) = 53.1°$

b. Show how you could use the cosine ratio (CAH) to find the measure of angle θ to the nearest tenth of a degree.

$\cos\theta = \frac{adj}{hyp} = \frac{3}{5}$ $\theta = \cos^{-1}\left(\frac{3}{5}\right) = 53.1°$

c. Find the ordered pair for point B'.

(−3, −4)

d. Find the measure of the angle of rotation β to the nearest tenth. Show how you found this angle measure.

$180° + 53.1° = 233.1°$

e. Which of the following statements is true: $\sin\theta = \sin\beta$ or $\sin\theta = -\sin\beta$? Explain why this is the true statement (hint: you might use the fact that $\sin\theta = \frac{y}{r}$).

True statement: $\sin\theta = -\sin\beta$

The sign of sine depends on the sign of the y-coordinate. y is positive in the 1st quadrant and negative in the 3rd quad

f. Which of the following statements is true: $\cos\theta = \cos\beta$ or $\cos\theta = -\cos\beta$? Explain why this is the true statement (hint: you might use the fact that $\cos\theta = \frac{x}{r}$).

True statement: $\cos\theta = -\cos\beta$

The sign of cosine depends on the sign of the x-coordinate. x is positive in the 1st quad and negative in the 3rd quadrant.

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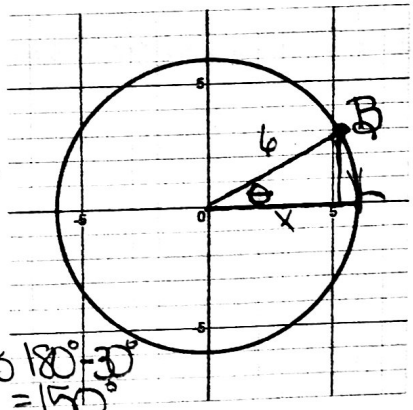
2. The diagram at the right shows the circle whose equation is $x^2 + y^2 = 36$. The measure of angle θ (in standard position) is 30°.

$r^2 = 36$ $r = 6$



a. Find the ordered pair for point B, which lies on the terminal side of angle θ, to the nearest tenth. Show how you found this ordered pair.

$\cos 30° = \frac{x}{6}$ $5.2 = x$ $\sin 30° = \frac{y}{6}$ $3 = y$ ordered pair (5.2, 3)

$6\cos 30° = x$ $6\sin 30° = y$

b. Use symmetry to name a different angle β such that $0° \le \beta \le 360°$ and $\sin\beta = \sin\theta$. Explain how you found β.

Sine is positive in quadrants 1 & 2. The ∠ in the 2nd quadrant that is 30° from the x-axis is 180° − 30° = 150°

c. Use symmetry to name a different angle α such that $0° \le \alpha \le 360°$ and $\cos\alpha = \cos\alpha$. Explain how you found α.

Cosine is positive in quadrants 1 & 4. The ∠ in the 4th quadrant that is 30° from the x-axis is 360° − 30° = 330°

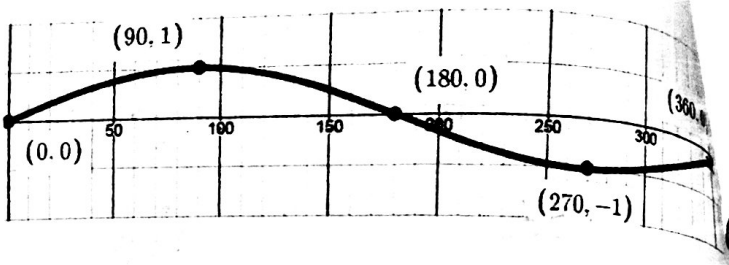
d. Which of the following statements is true: $\sin\theta = \sin(-\theta)$ or $\sin\theta = -\sin(-\theta)$? Explain why this is the true statement

True statement: $\sin\theta = -\sin(-\theta)$

Rotate clockwise when the angle is negative. ∴ The terminal side of angle $-\theta$ is in the 4th quadrant

Sine is positive in quadrant 1 (y is positive)
Sine is negative in quadrant 4 (y is negative)
Therefore, the sine values are opposites.

3. The graph of $f(x) = \sin x$ is given to the right.



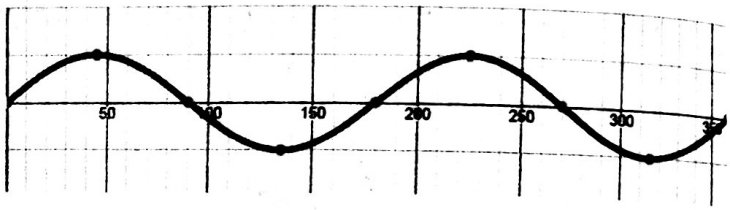
5.

Match the following equations with the following graphs. Explain how the graph was transformed from the parent graph of $f(x)$.

b $g(x) = 2\sin x$

The graph is stretched vertically by a factor of 2 (∴ the amplitude is greater)

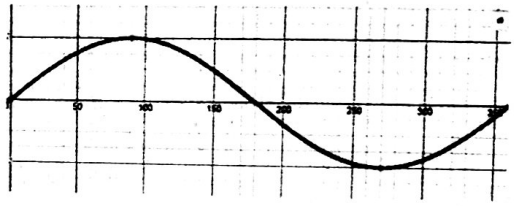
a.



c $h(x) = \sin x + 2$

The graph is translated up 2 units (∴ the midline is shifted up)

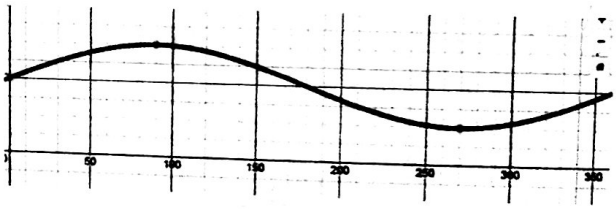
b.



a $j(x) = \sin(2x)$

The graph is shrunk horizontally by a factor of ½ (∴ the period is ½ as long)

c.



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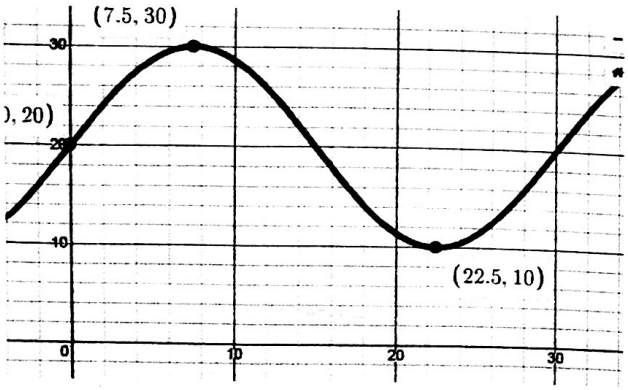
4. **Given the graph of $f(x)$, determine the following characteristics and write the equation.**



Amplitude: 10 "a" $\frac{\text{max}-\text{min}}{2} = \frac{30-10}{2} = 10$

Period: 30 "d"

Midline: $y = 20$ $\frac{\text{max}+\text{min}}{2} = \frac{30+10}{2} = 20$

Equation: $y = 10\sin(12x) + 20$ ← d

↑ a

↑ $b = \frac{360}{\text{period}} = \frac{360}{30} = 12$

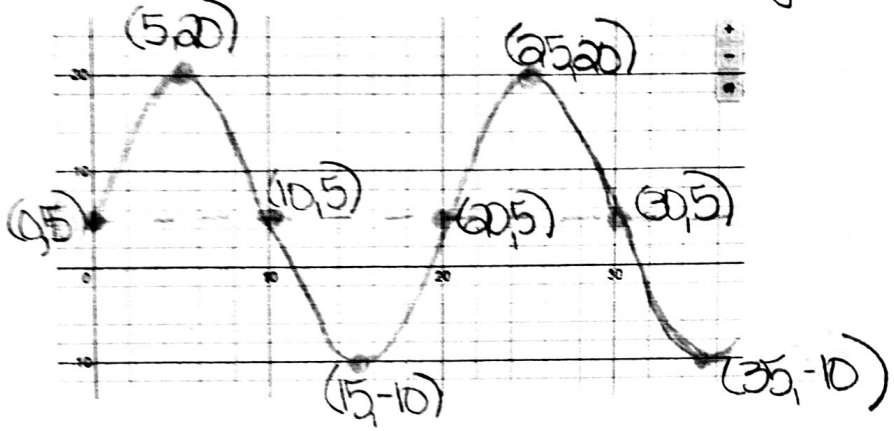
a b d

5. Given $f(x) = 15\sin(18x) + 5$, determine the following characteristics and graph:

"a" "d"

Amplitude: 15 Period: 20 Midline: $y=5$

$\text{Period} = \frac{360}{b} = \frac{360}{18} = 20$



How would the graph of $f(x)$ be different if the leading coefficient was -15 instead of 15?

The graph would be reflected over the midline.
∴ The graph would start at a middle height on its way down.

a b

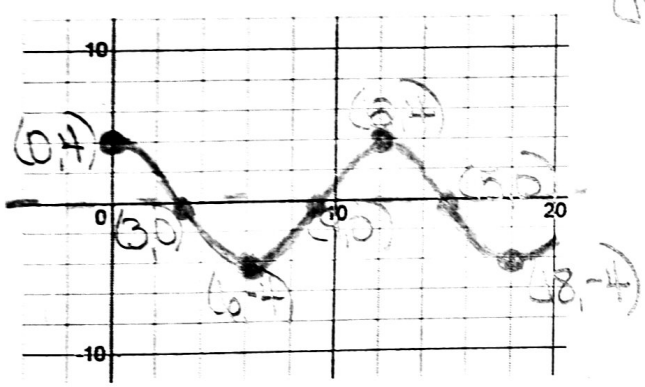
6. Given $g(x) = 4\cos(30x)$, determine the following characteristics and graph:

"a"

Amplitude: 4 Period: 12 Midline: $y=0$

(No constant was added/subtracted)

$\text{Period} = \frac{360}{b} = \frac{360}{30} = 12$



7. A Ferris Wheel whose maximum height is 160 feet has a diameter of 150 feet. The wheel completes one rotation every 15 seconds. Assume the rider is sitting in the car that is in the farthest right position when the wheel begins to turn in a counterclockwise direction.

$\text{radius} = \frac{\text{diameter}}{2} = \frac{150}{2} = 75$

a. What is the radius of the wheel? 75 ft

b. What is the period? 15 sec

$\text{angular speed} = \frac{360°}{15\text{sec}} = 24°/\text{sec}$

c. What is the angular speed in degrees/second? 24°/sec

Height of center * d. What is the height of the midline? 85 ft

max = 160
Diameter = 150
∴ Min = 10
$\text{Midline} = \frac{\text{max}+\text{min}}{2} = 85$

e. Write an equation of a sine function that models the height of the rider.

$f(x) = 75\sin(24x) + 85$

$b = \frac{360}{\text{Period}} = \frac{360}{15} = 24$

f. How high above the ground will the rider be after 5 seconds?

$x=5 \quad f(5) = 75\sin(24\cdot 5) + 85 = 149.95$

Min is 10 ft off the ground ~~+10~~ ~~160 ft~~

8. A 10" pizza is divided into 8 equal slices.

a. What is the measure of each central angle in degrees? 45°

$\frac{360°}{8} = 45°$

b. Convert the angle measure into radians. Show your work. $\frac{\pi}{4}$

$45° \cdot \frac{\pi}{180°} = \frac{\pi}{4}$

c. What is the arc length for 3 slices of pizza? Show your work.

$\frac{3(45°)}{360°} \cdot 2\pi(5) =$ (radius)

$\frac{15\pi}{4}$" or 11.78"

d. If a 12" pizza was also divided into 8 equal slices, would the measure of each central angle be different or the same as it was for a 10" pizza? The measure of each central ∠ would be the same (still $\frac{360°}{8}$)

Would the arc length for 3 slices be different or the same?

The arc length would be different because the arc length is proportional to the length of the radius.

(would be $\frac{3(45°)}{360} \cdot 2\pi(6)$)